

Heitor P. de Moura Filho

The Slave Population in Pernambuco, Brazil, 1560-1872: A Macrodemographic Reconstruction

Abstract

This paper exposes a demographic model to represent the evolution of the population of African slaves brought to the province of Pernambuco, Brazil, from 1560 to 1872, including their descendants. The main conclusion at this stage of the research is the confirmation that realistic combinations of mortality and fertility parameters, in a spreadsheet-based simulation, succeed in generating populations of size and distribution by age and sex that fit with those of Brazil's first nationwide census, in 1872. Such combinations of parameters also reinforce the thesis that the slave trade was the main motor for the growth of the slave population and its descendants in that region.

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The Slave Population in Pernambuco, Brazil, 1560-1872: A Macrodemographic Reconstruction¹

This article presents a demographic model for the development of the population of African slaves brought to a province of Brazil, Pernambuco, from 1560 to 1872, including their descendants. We make use of two essential sets of historical data. The time series of African slaves disembarked in Pernambuco, researched and organized by Daniel Domingues da Silva and David Eltis (2008), constitutes the main demographic flow to influence the population of this province. Data from the Brazilian census of 1872, adjusted in its age distribution, supplies the end-of-period reference. The model reconstructs populations coherent with both sets of data, through compatible combinations of mortality and fertility parameters.

The main conclusion at this initial stage of research is the confirmation that certain realistic combinations of mortality and fertility parameters, within the wide intervals discussed in the literature, do, in fact, generate populations with size and – more importantly – with the sex and age distributions registered in the first nation-wide census for Brazil, in 1872. This confirmation reinforces the thesis that the slave trade, in opposition to natural demographic increase, was the great motor for the growth of the slave population and its descendants in that region.

The difficulties in bringing together sufficient information for a more detailed demographic analysis of Brazil before 1872 are well known.² In comparison to other Brazilian provinces, Pernambuco before 1872 lacks complete and trustworthy data on most aspects of demographic interest. The provincial population surveys before the first nation-wide census were notoriously incomplete, according to Jerônimo Figueira de Mello (1979), the most important organizer of such information. Furthermore, there still do not exist, for Pernambuco, monographic studies based on ecclesiastic and notarial sources in the volume available for other provinces and captaincies, which would provide a substitute for census reports. In recent years, however, new research, now incorporated in the *Transatlantic Slave Trade Database*, has unearthed many sources for the demographic flows of greatest importance in forming the African-descendant population of Pernambuco: that is, the import slave trade (Florentino, Ribeiro et al. 2004; Silva 2004; Silva & Eltis 2008) and the internal slave trade (Barbosa 1995).³

Even error-prone and incomplete data, however, may be of use, if adequately analyzed and adjusted, as proposed by Giorgio Mortara:

In a country which offers perfect statistics for population flows, . . . the demographic census has great importance. . . . In a country which does not offer acceptable statistics for population flows, the importance of the demographic census is even greater, as it allows knowing approximately, at least in

part, some data fundamental to these flows, if one is not afraid of recurring, prudent and plausible hypotheses.’ (Mortara, 1940)

In a pioneering text, Mortara (1941) identified the existence of various types of errors in the Brazilian censuses and proposed their correction by methods that superimpose, on the data registered in the census, a combination of general tendencies and long-term demographic rates.⁴ One of the methodologies that he proposes for carrying out such corrections is the application of life-table coefficients to the data, so as to obtain age distributions similar in structure to the analyzed populations, but with age distributions compatible with the progression of the mortality observed in Brazil. We adapt this method to correct the age distribution of the population of descendants of Africans in 1872, generating the age distribution to be used as the target for a modeled population.⁵

The gathering of data from primary sources is, no doubt, the essential stage in increasing our knowledge of the historical demography of Brazil. To use such new data, we must also bring the critical analysis—and its consequent adjustments—of erroneous census data, as advocated by Mortara. We believe, however, that there is another methodological path, complementary to these first two, and as yet little explored in Brazil, which is the mathematical modeling of demographic data.

We see as the main advantages of demographic modeling the possibility of evaluating, in sequential steps, the consistency, in relation to data already known, of data that emerge from new research. There is a further advantage, within certain information contexts, of generating missing data and indicators otherwise impossible to calculate directly from the available data. This capacity of going “beyond” historical data is made possible by the logic of the demographic equation, rigid guardian of the plausible limits in quantitative matters regarding population. The model described here follows such a research program, presenting in preliminary format the results which associate two of the main sources for the historical demography of Pernambuco: the series of imported Africans by Silva & Eltis (2008) and the data of the Census of 1872 (Brasil 1872-76). We need to emphasize that our main objective here is to defend this form of combined empirical-analytic study; we are less concerned, at this point, with the specific historical interpretation of results.

The Model

Variables. The model, detailed in the Appendix, calculates the size of the population of African-descended inhabitants of Pernambuco. This population is divided into sub-populations by birthplace: “Africans” born in Africa and “Brazilians” born in Brazil, all of original African ancestry through the female line.⁶ Africans and Brazilians are each accounted by sex and by 101 age groups, constituting 404 sub-populations, all accompanied during the period from 1560 to 1872, that is, for 313 years, making up 126,452 population sizes for each combination of parameters. To obtain these totals, the model calculates 1,080 annual demographic events for each year:

- a) deaths in each of the 404 sub-populations;
- b) immigration into each of the 202 African sub-populations;
- c) births to African and Brazilian mothers in each of the 2 x 35 fertile age groups;
- d) emigration from each of the 404 sub-populations.

The model was programmed in Excel spreadsheets. Instead of the commonly-used 5-year chronological or age groups, we opted for annual periods throughout, due to ease of computation. This choice does not affect the results.

Parameters. The parameters, informed *a priori* and which may be altered to analyze alternative results, are the following age distributions:

- a) age composition of arrivals through the African slave-trade (one distribution for men and another for women);
- b) distribution of age-specific mortality rates for recently-arrived Africans, applied only in their first year in Brazil;
- c) distribution of age-specific mortality rates for Africans (from the second year on) and for Brazilians;
- d) distribution of age-specific fertility rates, during fertile ages from 15 to 49, normalized to one offspring during the course of a virtual life;
- e) age composition of emigrants, that is, a ‘propensity to emigrate’, by age group and sex, normalized to a total of one emigrant per year.

The following series of historical (or estimated) data for each year of study, from 1560 to 1872 are also parameters:

- f) total slaves imported (the Silva & Eltis series);
- g) total fertility rate for Africans and for Brazilian mothers;
- h) emigration rate for the total population.

The masculinity index of arrivals and of births in general are the parameters that complete the distributions and series mentioned above.

Age and fertility distributions here were calculated from continuous functions across all age groups (0 to 100), built from a subjective composition of beta and gamma functions, fine-tuned to obtain certain characteristics, such as a certain life expectation at birth. They are normalized to unity as discrete functions (i.e. the values for the 101 age groups add to one). Mortality distributions are adapted from Bulhões de Carvalho’s life tables for 1920, to approximate age distributions of the 1872 Census (Bulhões de Carvalho 1928). No attempt was made to ‘soften’ the irregularities in these distributions. Parameters which are desired to be fixed at certain values at certain moments in time (say e_0 at initial and final modeling years) may be interpolated during intermediate years between the fixed values, so that discontinuities are avoided as much as possible.

Adjustment of the Model

Through its parameterized design, the model is able to represent—given trustworthy information—most of the synchronic characteristics associated with population in general and specifically to populations of slaves and their descendants, such as differential mortality curves and age composition of first-generation slaves. It also includes, diachronically, the possibility of adjusting results to chronological data, attempting to reproduce correctly the fluctuations in slave-trade arrivals, and mortality or natality due to wars, revolts, droughts or epidemics. We have kept these historical “specificities” to a minimum in order to emphasize the main demographic forces at work. The import of Africans is the principal historically-determined series. The annual proportion of slave exports from Pernambuco has been estimated by its long-term tendency.

For any combination of parameters, one may recalculate the model so that the final populations (of men and women, both Brazilians and Africans) correspond to the totals established in the Census of 1872 (or to the populations obtained from adjustment to these totals). Furthermore, distributions of specific mortality rates have been sought which approximate the age composition at the end of the modeled period, in 1872. The most trying adjustments were, no doubt, designed to obtain these specific mortality curves, which produced a final population with the same age distribution as shown in the census. Given such age distributions, the adjustments for size were carried out by subjective changes to total mortality or total fertility levels, “guiding” the spreadsheet instruments to plausible solutions. More detailed studies could examine the effect of variations in a series of “propensities to emigrate” and in other parameters of the model, as well as analyzing the sensitivity of results to changes in parameters.

The End-population of the Model

We consider that the population to be modeled is that constituted of African slaves brought to Pernambuco, together with their descendants. To allow this population to be modeled separately from the total population of Pernambuco, we only consider as “descendants” of slaves the offspring of an African mother or of a female descendant of an African mother, independently of the father, whether he belongs or not to this population. This criterion excludes from the model only one group: children with a non-African or non-African-descendant mother, fathered by Africans or their descendants. To include this group in the model would require modeling the entire population of Pernambuco.

We decompose the population of African or African descendants in 1872 into the following groups:

- a) African slaves who remained in Pernambuco;
- b) freed African slaves who remained in Pernambuco;
- c) Brazilian descendants of African slaves who remained in Pernambuco as slaves;
- d) Brazilian descendants of African slaves who remained in Pernambuco as freed slaves;
- e) Brazilian descendants of African slaves who were born free and remained in Pernambuco.

The categories employed to classify the population in the Census of 1872, however, use another decomposition of total population, based both on social condition (free and slave) and on color or ethnic origin (white, black, mulatto and “caboclo”), as shown in Figure 1.⁷

CENSUS CATEGORY \ MODEL CATEGORY	Free whites	Free caboclos	Free mulattos	Free blacks	Black slaves	Mulatto slaves
African slaves					Correctly included	
Free Africans				Correctly included		
Brazilian slaves					Correctly included	Correctly included
Freed Brazilians	Incorrectly excluded	Incorrectly excluded	Correctly included	Correctly included		
Free Brazilians descending from Africans	Incorrectly excluded	Incorrectly excluded	Correctly included	Correctly included		
Free Brazilians not descending from Africans nor from native Indians	Correctly excluded					
Free Brazilians not descending from Africans, and with native Indian ascent		Correctly excluded	Incorrectly included			
	EXCLUDED		INCLUDED			

Figure 1. Cross-combinations of census classification and model classification

The cross-identification of these two classifications, thus, cannot be unambiguously defined. The census categories “slaves” and “free blacks” can be included without difficulty in the modeled population of “descendants of slaves.” When excluding the categories of “free whites” and “caboclos” and including that of “free mulattos,” however, we incur small errors as these categories present small subsets that are incorrectly excluded or incorrectly included in the population being modeled. In practical terms, we will build the end-population of 1872 from the census categories “free mulattos,” “free blacks,” “black slaves,” and “mulatto slaves,” incorrectly leaving out the following subsets:

- a) freed and free Brazilians descendants of Africans who might have been classified as “caboclo” in the census; and
- b) freed and free Brazilians descendants of Africans who might have been classified as “white” in the census.

We also incorrectly include the subset of free Brazilian not descending from Africans, though classified with native Indian ascent and which might have been classified as “free mulattos” in the census. Given the possible level of precision in a macrodemographic analysis of this type, we believe that these subsets do not have numerical relevance.

The Age Distribution of the End-population in the 1872 Census

As it is an essential parameter for the correct adjustment of the model, the concluding age distribution (in 1872) requires detailed analysis. We have opted, as a working hypothesis, to generate regular age distributions that maintain the registered total size of the population between 0 and 10 years of age. This procedure, which is close to that employed by Mortara, supposes that, though all children of these age groups have been registered in the census, there occurred only errors due to change in their age.⁸ Furthermore, these actuarial curves presuppose that the population at the time did not present age distortions in consequence of specific events in the 100 previous years, which would have unequally affected certain cohorts and not others. We do know that the population of slaves in Pernambuco, during the nineteenth century, did suffer from such events as, for example, differential mortality between slave and free population in epidemic spurts. We will leave the analysis of such “dents” in the age distributions for another occasion.

Figure 2 displays three age distributions relevant to the African-descended population of Pernambuco. All have the same size of the 0-to-10-years old sub-population. They are the 1872 Census population; “A,” a population calculated per Bulhões de Carvalho’s life table; and “B,” another population calculated according to a life table with

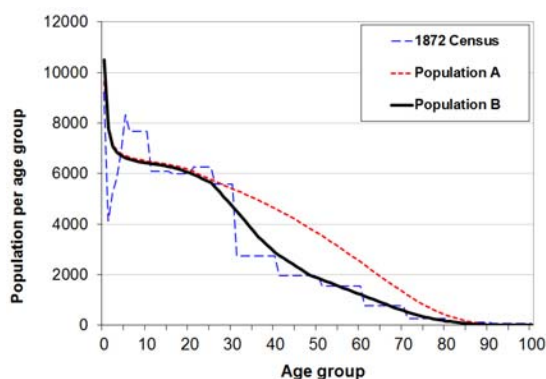


Figure 2. Three age distributions for males of African descent.
Sources: 1872 Census data: Brasil (1872-76); Life table “A”:
Bulhões de Carvalho (1928); life table “B”: author’s calculation.

greater mortality, built to approximate the census distribution. More precisely, the 1872 Census yields an age distribution of the registered population of “free mulattos,” “free blacks,” “mulatto slaves,” and “black slaves,” showing the total number of persons in each annual age group. *Population A* is presented as an age distribution of a

fictitious population calculated from Bulhões de Carvalho's life table for the 1920 Census, so that the total population from age 0 to 10 years will be equal to the registered population in the same age group. *Population B* is shown as the age distribution of a fictitious population calculated from its own life table, with the same size in the 0-to-10 age group, but with specific mortality closer to the apparent mortality of the 1872 Census.

If we accept the data of the 1872 Census as essentially correct, we could explain the mortality differential after age 25 existing between curve "A" and curve "B" through hygienic and medical progress that occurred between the 1872 and 1920 censuses, where curve "A" reflects the demographic characteristics of a population of the first two decades of the twentieth century, while curve "B" reflects a population half a century earlier.

The Data on Transatlantic Slave trade to Pernambuco

We have adopted, as our tally of immigrants, the slave trade annual series for Pernambuco presented by Daniel B. Domingues da Silva and David Eltis in *The Slave Trade to Pernambuco, 1561-1851*. The source used by these authors is the 2nd edition of the *Trans-Atlantic Slave Trade Database* (TSTD2, 2008), which compiles all currently known transatlantic voyages of slavers, as complemented by other sources covering smaller periods. This was the secondary source that aggregated the most detailed research on the matter.

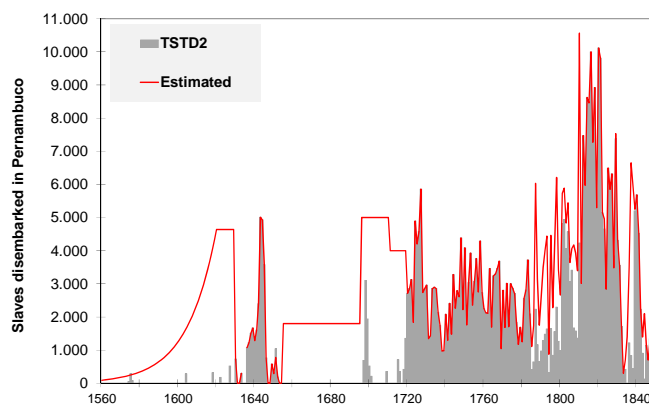


Figure 3. Slaves disembarked, Pernambuco, 1560-1872. Source: Silva & Eltis (2008).

Some Results

Development of the population. When examining the long-term evolution of the modeled masculine population, three periods clearly stand out regarding the relation between the African and Brazilian sub-populations.⁹

In Period 1, from 1560 to 1630, the sub-population of Brazilians grows (at an average of 4.86% per annum) basically due to the growth of the African population itself. That is, it represents essentially the first and second generation of Brazilians within an African population in rapid expansion (5.80% p.a.).

In Period 2, from 1630 to 1830, the Brazilian population continues to grow, but at a slower rhythm (on average 1.3% p.a.), while the African population oscillates in response to the spurts and retractions of the slave trade, always remaining below 100,000 individuals.

In Period 3, from 1830 to 1872, the Brazilian population maintains its increase at an even slower rhythm (0.6% p.a.), now entirely due to its own dynamics, while the African sub-population declines rapidly (at an average of -6.7% p.a.), reaching the census year representing only 1% of the joint population of Africans and their Brazilian descendants.

Period	Africans	Brazilians
1560 – 1630	5.80 %	4.86 %
1631 – 1830	0.26 %	1.31 %
1831 – 1872	-6.69 %	0.59 %

Table 2. Average annual growth rates of the modeled male population, Pernambuco, 1560-1872.
 Source: author’s model.

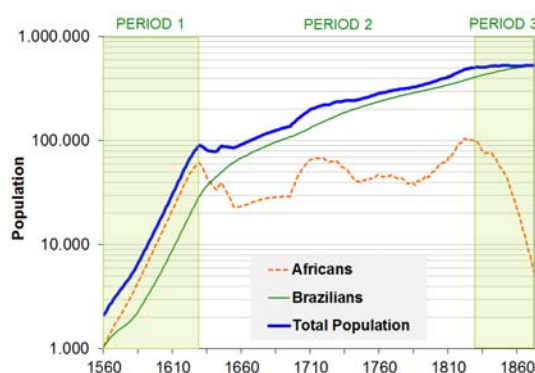


Figure 2. Modeled male population, by origin, Pernambuco, 1560-1872.
 Source: Author’s model.

Distribution of Africans and Brazilians. In another form of analysis of the same dynamics for the Brazilian and the African populations, we examine the proportion of Africans in the total population, a statistic frequently available in contemporary reports, and thus useful for comparisons between the model and specific sources. As expected, Figure 5 shows the increase in this proportion until 1630, corresponding to the growth in imports of Africans. From this date until 1830, the proportion of Africans declines as the total population grows and slave imports oscillate. After 1830, the proportion goes into a steady decline until becoming null.

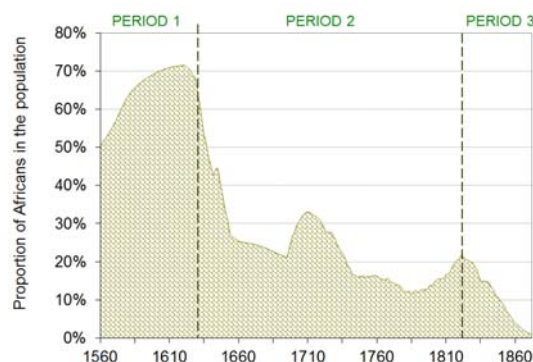


Figure 3. Proportion of Africans in the total population of Africans and African descendants, Pernambuco, 1560-1872. Source: Author’s model.

Development of the age distribution. Another indicator of interest for comparison of the modeled population and historical sources is the age distribution of sub-populations. We show here the development of this distribution for the African and the Brazilian sub-populations, which present quite differentiated characteristics. While the African group is renewed solely through immigration of groups with the same, young, age distribution, the Brazilian population evolves naturally, demonstrating its aging in the changing age distribution curve.

Figures 6 and 7, showing age distribution, are composed of discrete points indicating the number of individuals at each of 101 ages, from 0 to 100. Figure 6 shows the distribution of the African sub-population by one-year age groups in five different years: it thus shows the total size as well as the age distribution of this population. The group population over time shows growth and then decline, through the increase then reduction in the number of individuals, but remains relatively close to the age distribution of recently-arrived Africans. The equivalent age distribution curve for Brazilians “fattens,” in consequence of the progressive aging of this sub-population, through natural growth. These curves also expand upwards because of the increased numbers of Brazilians (Figure 7). Comparison of Figures 6 and 7 shows the relative size and the dramatically different age distribution of African and Brazilian sub-populations.

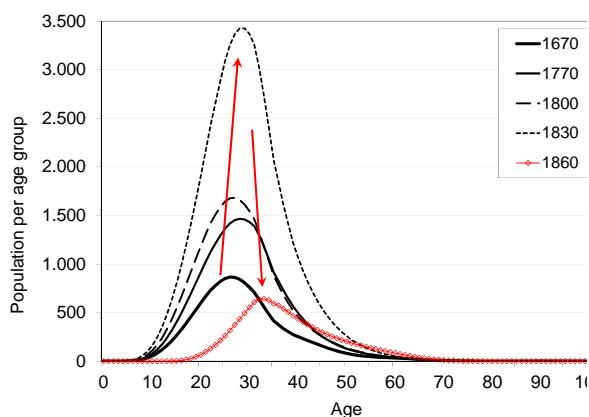


Figure 4. Age distribution of the African male population, 1670-1860. Source: Author’s model.

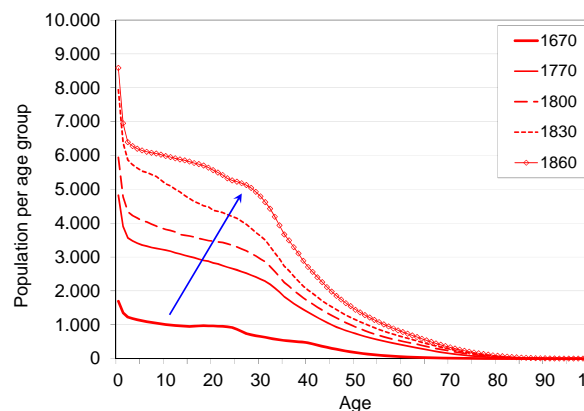


Figure 5. Age distribution of the Brazilian male population, 1670-1860. Source: Author’s model.

These distinct evolutions in age distribution can also be understood by examining the development of average age in the two sub-populations. While the average age of Brazilians stays below 25, consistent with a population undergoing natural growth, the average age of Africans remains—as expected—above the average age of recent arrivals, oscillating according to the volume of the slave trade and, once it stops, increasing rapidly (Figure 8). It is interesting to note that, in the period before 1630, when the main cause for growth of the Brazilian sub-population was the growth of the African sub-population itself (and thus of the number of potential mothers of Brazilians), this rapid growth made the Brazilian population younger, as it increased the number of births in Brazil.

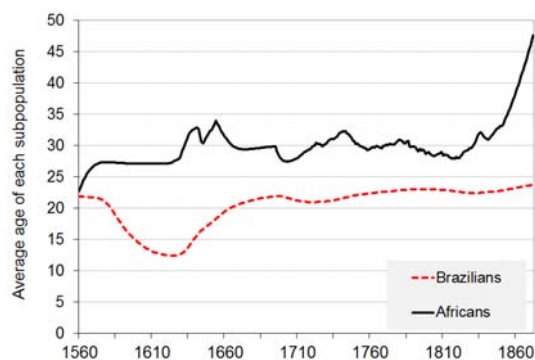


Figure 6. Average age of Brazilian and African sub-populations. Source: Author’s model.

Mortality

The sub-population-specific mortality curves (Figure 9) were built by successive approximations from the life tables produced by Bulhões de Carvalho (based on the 1920 Census), so that the final age distribution of the African and Brazilian sub-populations were close to the adjusted age distribution in the 1872 Census (Figure 2). The small irregularities in these curves originate from Bulhões de Carvalho’s tables. As these mortality curves are kept constant throughout the period of modeling, the only factor to alter crude mortality rates is the evolution of age distribution of the sub-populations, which can also be followed from their average age.

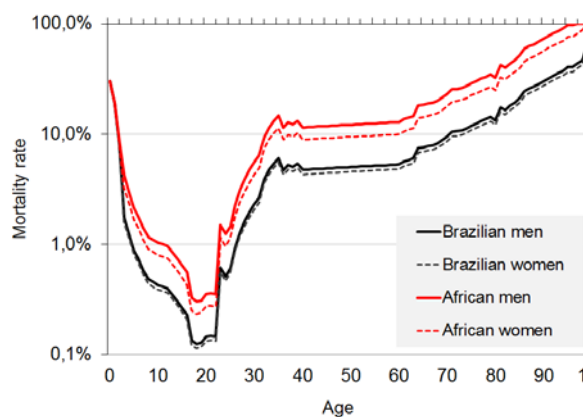


Figure 7 – Age-specific mortality rates, by sub-population (semilog scale). Source: Author’s model.

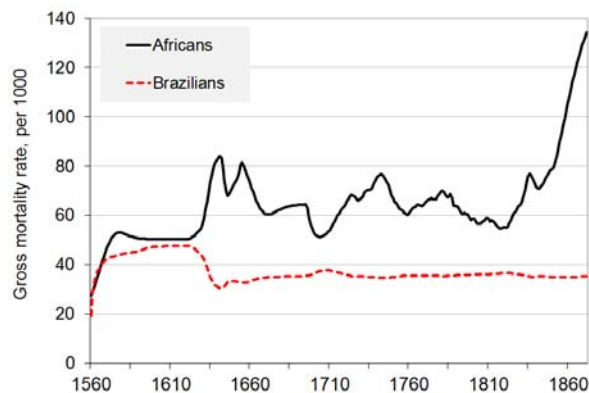


Figure 8 – Crude mortality rates, by sub-population. Source: Author’s model.

Period	Africans		Brazilians	
	CMR	Av. age	CMR	Av. age
1560-1630	48.9	26.9	44.5	16.7
1631-1830	64.6	29.9	35.3	21.1
1831-1850	73.5	31.7	35.3	22.5
1851-1872	108.7	39.8	35.0	23.3

Table 3. Crude mortality rates (CMR – per 1000) and average age (years) of the modeled male population, Pernambuco, 1560-1872. Source: Author’s model.

As an example of comparative statistics, among the many available in the literature, we can mention Eschwege’s comments, in 1815, on estimated average age at adult death in Minas Gerais, as reported by Pedro Carvalho de Mello (1977 p.108): whites, 28.3 years; mulatto slaves, 60.0; free mulattos, 27.5; black slaves, 68.6; free blacks, 53.8, Indians, 37.0. The compilation of statistics for comparison with model results is an important step to follow this preliminary modeling.

Nativity and Fertility

Within the combination of parameters chosen for this first test of the model, the progress of natality and fertility presents similar behavior for mothers of each birthplace, clearly influenced by the general age composition of each sub-population. Both indicators become stabilized at higher rates for Brazilian mothers than for African mothers, as should be expected from the proportionality between their respective total fertility parameters (African women had some of their children before reaching Pernambuco). The fluctuation in the rates for Africans is essentially due to the changes to the age distribution of fertile women brought about by oscillating imports of Africans. For Brazilian mothers, the rates start from average values corresponding to those of the hypothetical initial population, being radically reduced in the first decades of the model as the Brazilian population grows, with increase in the number of children. It then stabilizes, as this sub-population reaches a more stable age composition.

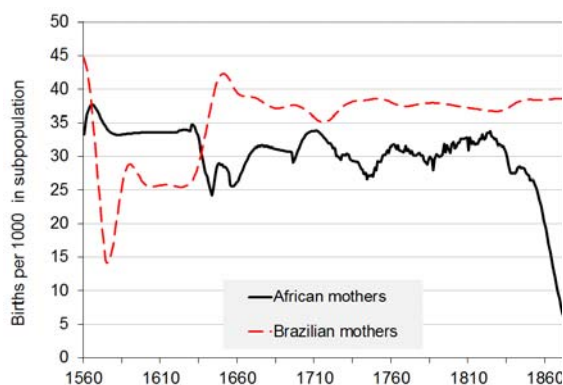


Figure 9. Natality (births per 1000 individuals in each sub-population). Source: Author’s model.

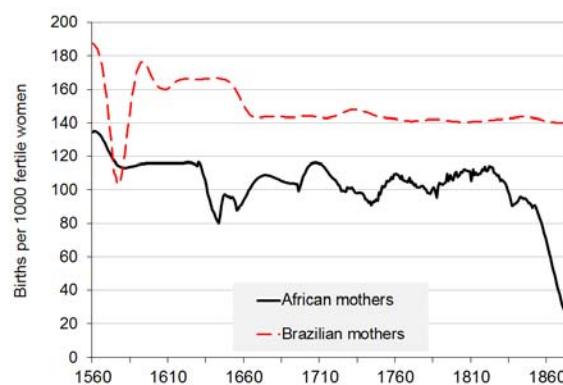


Figure 10. Fertility (births per 1000 fertile women in each sub-population). Source: Author’s model.

Components of Growth

As we have at hand, through the actual construction of the model, complete series of the number of individuals and of demographic events, it is a simple matter to analyze the components of variation in the total population, distinguishing the effects of immigration flows and of natural growth. We emphasize the necessity of defining and demonstrating with care this type of indicator, as the “migration effect” is not restricted to the population entering year after year. Migration, in addition to its immediate influence, implies changes to successive generations born (in Pernambuco or outside of it) from migrating mothers. We have shown this effect specifically in relation to the initial increase in the birth rate of Brazilian mothers, due almost exclusively to the increase in immigration of African women, future mothers of Brazilian mothers. We define this indicator here in its simplest form: the total number of net immigrants in a period, in relation to the average population of that period.

Until the beginning of the eighteenth century, we see important migratory effects, which are limited, in the next decades, to rates below 20 per 1000. The natural growth component is negative during almost all the period of study – due to the combined effect of a small percentage of women and a strong mortality (Figure 13) – and only presents an effective growth tendency as of the 1860s, as shown in the detail on the right side (Figure 14).

Obtaining this type of serial indicators is a result only possible with modeled populations, which again emphasizes the advantages of such a methodological approach.

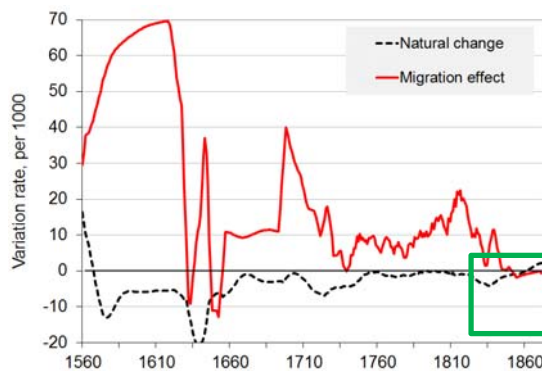


Figure 11. Variation rates for the modeled population (1560-1872). Source: Author’s model.

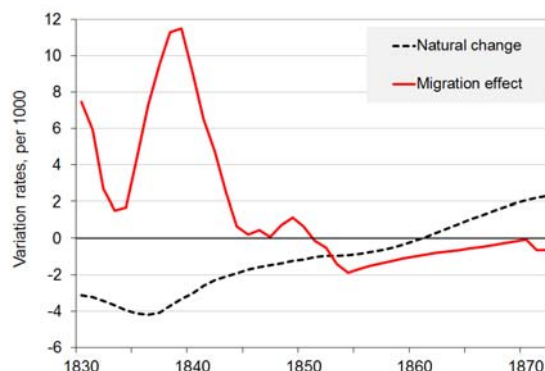


Figure 12. Variation rates for the modeled population. Detail of the 1830-1872 period.

Final Comments

In this first approximation, we do not examine systematically the consequences of the various possible logical and historical combinations of parameters. It has been our intention to show the general strength of demographic models combined with empirical data, which permits the analysis of long-term indicators calculated in coherent fashion with whatever historical data we may have at hand. The practical and analytic possibilities of such an approach for regions and periods plagued by lack of trustworthy data are clear.

We see future research progressing in three main directions: obtaining greater technical precision for the model as to the representation of demographic events; increasing the empirical basis to support the choice of its various

parameters; and producing sensitivity analysis of these parameters together with an analysis of the historical demographic indicators available in the literature, so as to increase its capacity for historical interpretation.

This initial modeling attempt can be technically and empirically improved in various ways, which we intend to pursue in future versions. We mention some of these possible improvements below:

- a) The age distribution of the end-population can be better analyzed and defined in relation to the degree of maintaining its irregularities and in relation to population totals. Modeling of the total population of a region and even a country should also be attempted.
- b) Other life-tables (such as G. Mortara's for 1870-1890 and E. Arriaga's for 1872 and 1900) should be analyzed in relation to Bulhões de Carvalho's tables based on the 1920 Census, to define the most adequate mortality standard for the model.
- c) Given the lack of knowledge about actual mortality rates, the analysis of statistical relations between specific mortalities for Africans and Brazilians, as well as between those for men and women, which are easier to confirm empirically, will lead to corresponding revisions of model parameters.
- d) Given the distinct social and demographic conditions of each of the macro-periods, specific mortality curves and levels for each of these should bring added precision to the estimates.
- e) The age distribution of annual arrivals, due to its importance in shaping the modeled population, merits a sensitivity analysis in relation to average age (reflecting concentration in teenagers, young adults or mature adults). For a more historically convincing model, this age distribution should also accompany the changes in the conditions of the slave trades that occurred in the macro-periods.
- f) The annual emigration rate should be fine-tuned in relation to the two main internal slave-trade export periods from Pernambuco: during the Minas Gerais gold boom in the early eighteenth century and in the period after 1840, when Rio de Janeiro and São Paulo coffee plantations imported large numbers of slaves from the Northeast.
- g) A more precise definition of plausible fertility levels can be sought through the analysis of indicators more commonly found in existing statistics, such as children per woman.
- h) Specific events with major demographic effects, such as epidemics, droughts and wars or revolts, quite frequent in the population history of Pernambuco, should have their consequences on deaths and births estimated.
- i) As a more advanced exercise, once the various demographic parameters can be more strongly defended, the development of a modeled population could be compared to contemporary population estimates, eventually leading to a reappraisal of the quality of existing population statistics and to the proposal of strong estimates for missing or incorrect data.

It must be emphasized that the main advantage of such a model is its repeatability with successive—and hopefully more precise—data sets. Its aim at the limit would be accounting for the diverse oscillations in demographic events—births, deaths and migrations—during the course of the region's population history.

APPENDIX – DESCRIPTION OF THE MODEL

1. General Characteristics

1.1. Generic notation

SYMBOL	DESCRIPTION
$X(t,i)$	Population variable X , referring to the end of period t , associated with the cohort of complete age i —or demographic flow variable X , referring to the flow during period t , associated to the same cohort
$X^A(t,i)$	Ditto, referring only to the sub-population of Africans
$X^B(t,i)$	Ditto, referring only to the sub-population of Brazilians
$x(i)$	Age-specific rate (of mortality or fertility) associated to cohort of complete age i ; or proportion of a population in an age group between complete ages i (included) and $i+1$ (excluded)
$x^A(i)$	Ditto, referring only to the sub-population of Africans (^A) or Brazilians (^B)
P or p	Variable referring to the total population or one of its sub-populations
D or d	Variable referring to deaths
N	Variable referring to births (<i>nascimentos</i>)
I	Variable referring to immigration/imports of African slaves
E or e	Variable referring to emigration/exports of slaves
F or f	Variable referring to the number of women in fertile age or to fertility rates
s	Proportion of men in a sub-population or demographic flow
r	Masculinity proportion in a sub-population or demographic flow
[P]	Indicates a parameter, given <i>a priori</i>
[V]	Indicates a variable, to be calculated by the model

1.2. Demographic variables. All modeled variables—sizes of populations and number of demographic events—are calculated from continuous functions in an Excel spreadsheet, without modeling of individual life cycles. Contrary to what occurs in models which generate individuals and simulate their life cycle from events and characteristics determined through probabilistic parameters, this methodology avoids the necessity of multiple repetitions of calculations (for example in a Monte Carlo analysis) to obtain averages, as all parameters already incorporate the probabilities of desired events, thus already representing averages with any size population.

Demographic events occurring in a certain year for a certain age group are considered as risks to the relevant population existing at the end of the previous year aged one year less. No intra-year effects are considered. The time index, t , runs from 1560 to 1872. The age index, i , runs from 0 to 100.

1.3. Initial population. The initial population of the model, which can be set at any desired size or composition, corresponds to that which would have existed at the end of 1559. It is divided by sex according to the same masculinity proportion and the same age distribution as the annual group of slaves arriving. This year was chosen as the year right before the beginning of the Eltis & Silva series for imports of African slaves. In this model it is considered to be null.

1.4. Descendants. The model considers as descendants of the modeled population the offspring of African women and of their Brazilian daughters, from fathers included or not in the modeled population. The descendants of men included in the model with women not included (white or native Indians) are not considered in the model, as keeping account of these would require modeling the entire population of Pernambuco. The modeled population is made up of all these descendants, without distinction of social condition (free, slave or freed) or of color or ethnical origin.

2. Imports of Slaves

2.1. Parameters and variables

SYMBOL	DESCRIPTION
$I(t,i)$	[V] Number of recently-arrived slaves, during the year t , with complete age i
$I(t,\bullet)$	[P] Number of recently-arrived slaves, during the year t
s^{RC}	[V] Percentage of men in the recently-arrived group
r^{RC}	[P] Masculinity proportion in the recently-arrived group
$p^{RC}(i)$	[P] Age distribution percentages of the recently-arrived of complete age i , which is maintained equal for all t

The number of total recently-arrived persons in each year, $I(t,\bullet)$, is a fundamental historical parameter, given by the time series estimated by Eltis & Silva.

2.2. Immigrants / recently-arrived slaves. This total of recently-arrived slaves is distributed by sex according to a fixed percentage for all ages and all years, s^{RC} . The totals for each sex are distributed by age groups according to the formulae:

a) for men:

$$[1] I(t,i) = I(t,\bullet) s^{RC} p^{RC}(i), \text{ where}$$

$$[2] s^{RC} = r^{RC}/(1+r^{RC}),$$

with r^{RC} measured in number of men per woman (and not as usually presented per 100 women).

b) for women:

$$[3] I(t,i) = I(t,\bullet) (1-s^{RC}) p^{RC}(i),$$

in both cases,

$$[4] \sum_i p^{RC}(i) = 1.$$

The age distributions $p^{RC}(i)$ are specific for men and for women. We avoided including this further distinction in the notation to make it unnecessarily dense.

3. Deaths

3.1. Parameters and variables

SYMBOL	DESCRIPTION
$D(t,i)$	[V] deaths of Africans and Brazilians, during year t , with complete age i
$D^A(t,i)$	[V] deaths of Africans, during year t , with complete age i
$D^B(t,i)$	[V] deaths of Brazilians, during year t , with complete age i
$d^A(i)$	[P] Specific rate of mortality for Africans between ages i (included) and $i+1$ (excluded)
$d^{RC}(i)$	[P] Specific rate of mortality for recently-arrived Africans between ages i (included) and $i+1$ (excluded)
$d^B(i)$	[P] Specific rate of mortality for Brazilians between ages i (included) and $i+1$ (excluded)

These specific mortality rates – $d^A(i)$, $d^{RC}(i)$ and $d^B(i)$ – vary according to sex and to the macroperiods (1560 to 1830; and 1831 to 1872).

3.2. Deaths of Africans. The number of deaths of Africans in each year is calculated on the population at risk at the end of the previous year, at the specific mortality rates referring to a cohort one year older, plus deaths of recently-arrived in the year of reference. The specific mortality rates for recently-arrived slaves, which were significantly higher than those for settled Africans and for Brazilians, are applied only in the year of arrival. In order to reflect this immediate mortality, post-arrival, to which this group would be subjected to for an average of 6 months, we consider 50% of these rates over the total number of recently-arrived slaves.

$$[5] D^A(t,i) = P^A(t-1,i-1) d^A(i) + I(t,i) d^{RC}(i)/2, \text{ with } i > 0.$$

For the initial age group, with less than one complete year, ($i=0$), the following formula is applied:

$$[6] D^A(t,0) = I(t,0) d^{RC}(0)/2.$$

The total number of deaths of Africans is equal to the sum of totals by age group:

$$[7] D^A(t,\bullet) = \sum_i D^A(t,i).$$

3.3. Deaths of Brazilians. The deaths of Brazilians in each year are calculated over the population at risk at the end of the previous year, at the specific mortality rates referring to a cohort one year older.

$$[8] D^B(t,i) = P^B(t-1,i-1) d^B(i), \text{ with } i > 0.$$

For the initial age group, with less than one complete year, ($i=0$), the following formula is applied:

$$[9] D^B(t,0) = N(t,\bullet) d^B(0)/2$$

The total number of deaths of Brazilians is equal to the sum of deaths by age groups:

$$[10] D^B(t, \bullet) = \sum_i D^B(t, i) .$$

4. Births

4.1. Parameters and variables. All individuals born in the model are Brazilians. It is possible to analyze separately the births to African and to Brazilian mothers.

SYMBOL	DESCRIPTION
$N(t, i)$	[V] Number of births, during year t , to mothers of complete age i
$N^A(t, i)$	[V] Ditto, births to African mothers (^A); ditto, to Brazilian mothers (^B)
$N(t, \bullet)$	[V] Total number of births during year t
s^N	[V] Percentage of men in the births of a year
r^N	[P] Masculinity proportion in births
$f^A(i)$	[P] Specific fertility rate for African women between ages of i (included) and $i+1$ (excluded), normalized to 1 per life cycle
$f^B(i)$	[P] Specific fertility rate for Brazilian women between ages of i (included) and $i+1$ (excluded), normalized to 1 per life cycle
$f(t, \bullet)$	[P] Total fertility rate, variable per year
$f^A(t, i)$	[P] Ditto, for African women (^A); ditto, for Brazilian women (^B)
$F(t, i)$	[V] Number of women of fertile age, with complete age i , at the end of year t
$F^A(t, i)$	[V] Ditto, for African women (^A); ditto, for Brazilian women (^B)
$F(t, \bullet)$	[V] Total number of women of fertile age at the end of year t

The masculinity proportion at birth is fixed for both Africans and Brazilians, and for all periods at 1,05. The specific fertility distributions are fixed in form, but may reflect annual variations in total fertility through the parameter (t, \bullet) .

4.2. Number of women of fertile age. The number of women of fertile age at the end of year t is the sum of women in the 15 to 49 age groups at this moment.

$$[11] F(t, \bullet) = \sum_i F(t, i) , \text{ with } 14 < i < 50.$$

Ditto, only for Africans $F^A(t, \bullet)$ or only for Brazilians $F^B(t, \bullet)$.

4.3. Specific fertility rates. For the calculation of total births, the curves of specific normalized fertility, $f^A(i)$ e $f^B(i)$, are compounded with total fertility, an annual parameter. All events which influence total births, such as the final parity of each woman or eventual reductions in the number of births due to epidemics or wars, are considered as incorporated in these fertility parameters. The fertility distribution corresponds to the number of births the average woman would have, during the course of her completed fertile life. It is applied to 100% of women living at

the end of each year. Multiplying these age-specific normalized rates by the total desired fertility rate for each year, we arrive at the required age-specific total fertility for that age group in that year.

4.4. Number of births. The number of births to mothers of each age group is the result of the multiplication of the age-specific normalized fertility rate, by the total fertility desired for that year, further multiplied by the number of fertile women in the age group.

$$[12] N(t,i) = f(i) f(t,\bullet) F(t-1,i-1).$$

The total number of births in year t is equal to the sum of births to mothers of all fertile age groups, during that year:

$$[13] N(t,\bullet) = \sum_i N(t,i) = \sum_i f(i) f(t,\bullet) F(t-1,i-1).$$

Ditto, only for African mothers $N^A(t,\bullet)$ or only for Brazilian mothers $N^B(t,\bullet)$.

5. Emigrants

5.1. Parameters and variables. The term “Emigrants” refers to the exit flow of free emigrants from Pernambuco and that due to interprovincial slave trade. Eventual immigrations of Brazilians back into Pernambuco (rare and historically most probably due to slaves or free African descendants entering from states further north than Pernambuco) are considered as already deducted from this net emigration rate, and are not treated separately.

SYMBOL	DESCRIPTION
$E(t,i)$	[V] Emigrants, during year t , with complete age i
$E(t,\bullet)$	[V] Total emigrant in year t
$E^A(t,i)$	[V] African emigrants, during year t , with complete age i
$E^B(t,i)$	[V] Brazilian emigrants, during year t , with complete age i
$e^A(i)$	[P] Age-specific normalized propensity to emigrate for Africans with age between i (included) and $i+1$ (excluded)
$e^B(i)$	[P] Age-specific normalized propensity to emigrate for Brazilians with age between i (included) and $i+1$ (excluded)
$e(t,\bullet)$	[P] Percentage of the total population at end of year $t-1$ to emigrate in year t

Specific emigration propensities for recently-arrived slaves, though not included in this version, would permit modeling flows of slaves destined to other states, such as occurred in the peak of the gold rush in Minas Gerais, when slaves imported to Pernambuco travelled inland (by foot) some 2,500km to reach the high-demand mining region.

5.2. Number of emigrants. Contrary to immigration through the slave trade, whose total can be estimated, emigration totals are calculated given a certain annual proportion of emigrants relatively to the total population then. The total number of emigrants by age group is calculated as the product of this total annual number of emigrants and the age-specific propensity to emigrate, $e(i)$ being the propensity to emigrate of individuals of complete age i , where

$$[14] \sum_i e(i) = 1 ,$$

and $e(t, \bullet)$ the proportion of emigrants in a certain year, relatively to the existing population at the end of the previous year, $P(t-1, \bullet)$, the annual number of emigrants in each age group is given by

$$[15] E(t, i) = e(t, \bullet) e(i) P(t-1, i-1),$$

where the parameters $e(i)$ and $P(t-1, i-1)$ refer specifically to one of each of the groups: Africans and Brazilian, men and women. The total number of emigrants is the sum

$$[16] E(t, \bullet) = \sum_i E(t, i).$$

6. Population

6.1. Parameters and variables

SYMBOL	DESCRIPTION
$P(t, \bullet)$	[V] Total population at end of year t
$P^A(t, \bullet)$	[V] Total population of Africans at end of year t
$P^B(t, \bullet)$	[V] Total population of Brazilians at end of year t
$P(t, i)$	[V] Total population at end of year t, with complete age i

6.2. African population. The African population is calculated as follows:

a) the initial population is null;

b) population at end of year t:

$$[17] P^A(t, \bullet) = \sum_{i>0} [P^A(t-1, i-1) + I(t, i) - D^A(t, i) - E^A(t, i)] + P^A(t, 0)$$

For the initial age group, with less than one full year, the following formula is applied:

$$[18] P^A(t, 0) = I(t, 0) [1 - d^{RC}(0)/2] .$$

6.3. Brazilian population. The Brazilian population is calculated as follows:

a) initial population: as desired (in this case, null);

b) population at end of year t:

$$[19] P^B(t, \bullet) = \sum_{i>0} [P^B(t-1, i-1) - D^B(t, i) - E^B(t, i)] + P^B(t, 0)$$

For the initial age group, with less than one full year, the following formula is applied:

$$[20] P^B(t, 0) = N(t, \bullet) [1 - d^B(0)/2] .$$

NOTES

¹ A previous Portuguese version of this paper was presented at the 2008 Congress of the Brazilian Association for the Study of Population (ABEP). We thank the editors and two referees for valuable comments and criticism, which we have incorporated in this text and will consider in future papers.

² Examples of opinions on this point: “Uncertainty as to the size of the Brazilian population at the beginning of the nineteenth century makes it difficult to establish a benchmark for analysis of trends in the period prior to 1872.” (Merrick & Graham 1979 p.26) “The known data for the demography of the Brazilian colonial past are, in general, fragmentary and of small credibility. Thus, we do not have an integral vision of its demographic structures and much less of the dynamics of its population.” (Balhana 1986 p.21)

³ Since this paper was written, the Transatlantic Slave Trade Database has been regularly updated. Data for disembarkations in Pernambuco available in July 2015 include 853,833 slaves versus the 681,756 used in this modeling, i.e. 10.0% more. This would probably result in an increase in the estimates of mortality arrived at in this version.

⁴ “As the examination of the age distributions of the population of Brazil in the various censuses has shown us that they suffer from serious errors, . . . the convenience of rectifying such distributions is evident, in order to enable their use, direct or indirectly, in the study of the demography of Brazil. The multiplicity and reciprocal interferences of the errors which push the census data away from the truth make extremely difficult any attempt at correction which approaches such errors separately. Furthermore, in many cases there is a lack of basis for such attempts. Thus, we believe that to reach a satisfactory correction of the age distributions it is necessary to search for a simultaneous and integral correction of the age distribution, through the application of a systematic process, which though valid only for Brazilian-born individuals counted in the censuses, seems adequate to our aim.” (Mortara 1941 p.39-40)

⁵ We do not make use of standard life-tables with incorporated positive natural growth, based on stable population techniques. In the model, natural growth is a consequence of desired fertility parameters, independent of mortality parameters.

⁶ In addition to the overall African-descended population and the two main sub-populations of Africans and Brazilians, the model and the discussion here also address smaller “sub-populations,” identified by sex and age within the African and Brazilian sub-populations. Note that the initial conditions of the model include zero “Brazilians” or Brazilian-born population.

⁷ “Caboclo” are descendants of native Indians, mixed either with whites or blacks. The census uses the term as a “catch-all” for individuals not considered in the “mulatto” category and also excluded both from that of “blacks” and of “whites.” This classification in the 1872 Census was even more subjective than in later censuses, based mostly on the census officer’s appraisal.

⁸ We believe that this hypothesis underestimates the population total, as the non-counting of children is a well-known census error.

⁹ “Population,” “masculine population,” etc. are always references to the population of Africans and their descendants, as described here. We do not discuss the total population of Pernambuco.

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